THE CHAOTIC GROWTH MODEL: INTERNATIONAL TOURISM

Vesna D. Jablanovic[1] (iD)

Abstract *The basic aims of this paper are: firstly, to create a relatively simple chaotic international tourism expenditure growth model that is capable of generating stable equilibria, cycles, or chaos; and secondly, to set up a relatively simple chaotic international tourism receipts growth model. This paper confirms stable growth of the international tourism receipts and expenditures in high-income countries, low & middle-income countries, lower-middle-income countries, middle-income countries, and upper-middle-income countries in the period 1995-2018*.

Keywords: *Expenditure, Receipts, Tourism, Growth, Stability, Chaos.*

1. INTRODUCTION

Tourism is one of the fastest-growing economic sectors. In 2018 tourism receipts amounted to \$1,480 billion, an increase of 4.4. percent. Tourism exports account for 7 percent of global trade in goods and services (\$1.7 trillion). In 2019, the most popular destinations were France, Spain, the USA, and China. International tourism is among the economic sectors most impacted by the COVID-19 pandemic. The United Nations World Tourism Organization (UN WTO) estimates a loss of 850 million to 1.1 billion international tourist arrivals, \$910 million to \$1.1 trillion in export revenues, and 100-120 million jobs (UNCTAD, 2020, pg.7).

This paper uses the elements of chaos theory (Jablanovic, 2019). Also, this paper confirms stable growth of the international tourism receipts (as the ratio to exports of goods and services) and the international tourism expenditures (as the ratio to imports of goods and services) in high-income countries, low & middle-income countries, lower-middle-income countries, middle-income countries, and upper-middle-income countries in the period 1995-2018.

2. THE CHAOTIC INTERNATIONAL TOURISM EXPENDITURE GROWTH MODEL

International tourism expenditures (E) are expenditures of international outbound visitors in other countries, including payments to foreign carriers for international transport. They may include those by residents traveling abroad as same-day visitors, except in cases where these are important enough to justify separate classification. Their share in imports (e) is calculated as a ratio to imports of goods and services (I), which comprise all transactions between residents of a country and the rest of the world involving a change of ownership from non-residents to residents of general merchandise, goods sent for processing and repairs, nonmonetary gold, and services (www.worldbank.org).

$$e = E / I \qquad (1)$$

where:

e – the ratio of international tourism expenditures (E) to imports of goods and services (I).

---
[1]   University of Belgrade, Faculty of Agriculture, Nemanjina 6, 11081 Belgrade, Serbia

We index e by t, i.e., write e_t to refer to the size at time steps $t = 0, 1, 2, 3, ...$ Now the international-al tourism expenditures growth rate is measured by the quantity already given corresponding to the expression:

$$\frac{e_{t+1} - e_t}{e_t} \tag{2}$$

It is postulated that the growth rate of the ratio of international tourism expenditures (E) to imports of goods and services (I) at time t should be proportional to $1 - e_t$, after introducing a suitable parameter γ

$$\frac{e_{t+1} - e_t}{e_t} = \gamma\,(1 - e_t) \tag{3}$$

Solving the last equation yields the growth model, i.e.,

$$e_{t+1} = e_t + \gamma\,e_t\,(1 - e_t) \tag{4}$$

or,

$$e_{t+1} = (1 + \gamma)\,e_t - \gamma\,e_t^2 \tag{5}$$

This model given by equation (5) is called the logistic model. For most choices of γ, there is no explicit solution for (5). Namely, knowing γ and measuring e_0 would not suffice to predict e_t for any point in time, as was previously possible. Lorenz (1963) discovered this effect – the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos. This difference equation (5) will possess a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point e_0 the solution is highly sensitive to variations of the parameter γ; secondly, given the parameter γ, the solution is highly sensitive to variations of the initial point e_0. In both cases, the two solutions are for the first few periods rather close to each other, but later on, they behave chaotically.

It is possible to show that the iteration process for the logistic equation:

$$z_{t+1} = \pi\,z_t\,(1 - z_t),\ \pi \in [0, 4],\ z_t \in [0, 1] \tag{6}$$

is equivalent to the iteration of the growth model (5) when we use the identification

$$z_t = \frac{\gamma}{\gamma + 1}\,e_t \tag{7}$$

and

$$\pi = 1 + \gamma$$

Using (7) and (5) we obtain

$$z_{t+1} = \frac{\gamma}{\gamma + 1}\,e_{t+1} = \frac{\gamma}{\gamma + 1}\,[(1 + \gamma)\,e_t - \gamma\,e_t^2] = \gamma\,e_t - \frac{\gamma^2}{\gamma + 1}\,e_t^2$$

Using (6) and (7) we obtain

$$z_{t+1} = \pi z_t (1 - z_t) = (\gamma + 1) \frac{\gamma}{\gamma + 1} - e_t (1 - \frac{\gamma}{\gamma + 1} e_t) = \gamma e_t - \frac{\gamma^2}{\gamma + 1} e_t^2$$

Thus we have that iterating (5) is the same as iterating (6) using (7). It is important because the dynamic properties of the logistic equation (6) have been widely analyzed (Li and Yorke (1975), May (1976)).

It is obtained that :
- For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$;
- (ii) For $1 < \pi < 3{,}57$ there exist fixed points the number of which depends on π;
- (iii) For $1 < \pi < 2$ all solutions monotnically increase to $z = (\pi - 1) / \pi$;
- (iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$;
- (v) For $3 < \pi < 4$ all solutions will continuously fluctuate;
- (vi) For $3{,}57 < \pi < 4$ the solution become „chaotic" which means that there exist aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates randomly over time, not settling down into any regular pattern whatsoever.

3. EMPIRICAL EVIDENCE

The main aim of this paper is to analyze the local growth stability of the ratio of international tourism expenditures (E) to imports of goods and services (I) in the period 1995-2018. in: high-income countries, low&middle income countries, lower-middle-income countries, middle-income countries, and upper-middle-income countries (see Figures 1-5). In this sense, it is important to use the logistic model (8):

$$e_{t+1} = \pi e_t - \gamma e_t^2 \qquad\qquad (8)$$

where:
e - the international tourism expenditure (as the ratio to imports of goods and services),
π - $(1+\gamma)$, γ – the constant of proportionality.

Now, model (8) is estimated (see Tables 1-5).
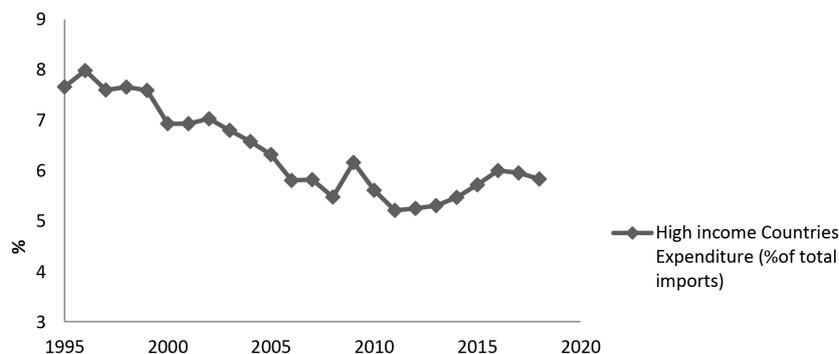
Figure 1. High-income countries, Expenditures (% of total imports), 1995-2018.
Source: https://data.worldbank.org/indicator/ST.INT.XPND.MP.ZS?view=chart

Table 1. The estimated model (8): High-income countries, 1995-2018

		R = 0.93318 Variance explained: 87.082%	
High-income countries		π	γ
	Estimate	1.06864	0.099957
	Std.Err.	0.07663	0.091512
	t(21)	13.94467	1.088348
	p-level	0.00000	0.288775

The expenditures moved from 7.6654 to 5.8421 (unit of measure: % of total imports) in the period 1995-2018 in high-income countries. According to the logistic equation, for $1 < \pi < 2$ the expenditures moved to $z = (\pi - 1) / \pi$, or $(1.06864 - 1) / 1.06864 = 0.06423$. According to (7.), the equilibrium value of the expenditures was $0.06423 / (0.099957 / 1.06864)$ or $0.06423 / 0.09354$ or 0.68666 or 5.4858 (unit of measure: % of total imports) in high-income countries in the observed period.
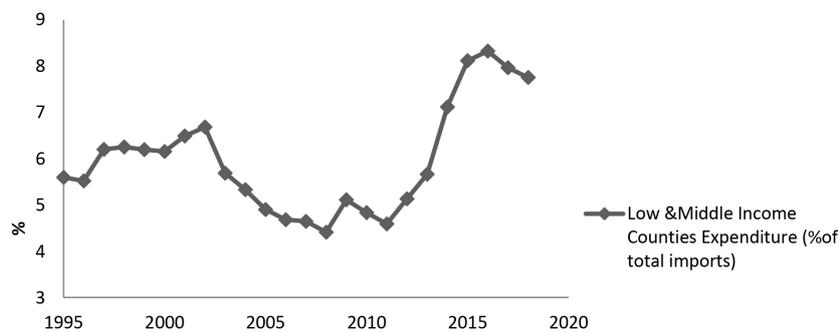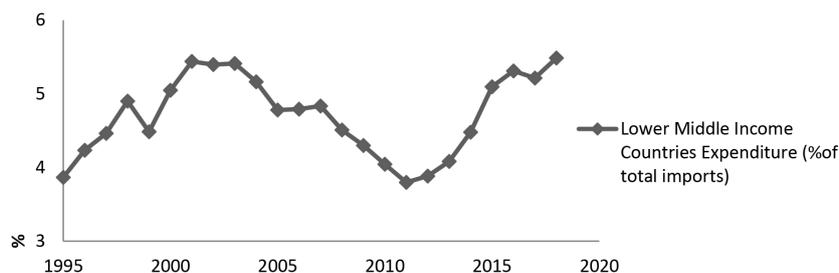

Figure 2. Low & middle-income countries, Expenditures (% of total imports)
Source: https://data.worldbank.org/indicator/ST.INT.XPND.MP.ZS?view=chart

Table 2. The estimated model (8): Low & middle-income countries, 1995-2018.

		R = 0.89689 Variance explained: 80.441%	
Low & middle-income countries		π	γ
	Estimate	1.08412	0.092879
	Std.Err.	0.10069	0.129867
	t(21)	10.76675	0.715185
	p-level	0.00000	0.482371

The expenditures moved from 5.6032 to 7.7564 (unit of measure: % of total imports) in the period 1995-2018 in low & middle income countries. According to the logistic equation, for $1 < \pi < 2$ the expenditures moved to $z = (\pi - 1) / \pi$, or $(1.08412 - 1) / 1.08412 = 0.0776$. According to (7.), the equilibrium value of the expenditures was $0.0776 / (0.092879 / 1.08412)$ or $0.0776 / 0.08567$ or 0.9058 or 7.5411 (unit of measure: % of total imports) in low & middle income countries in the observed period.
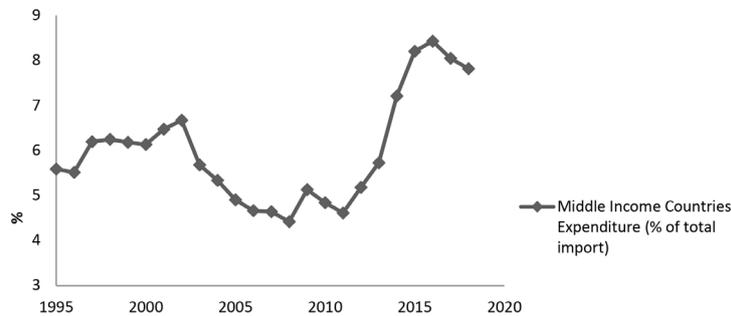

Figure 3. Lower middle-income countries, Expenditures (% of total imports).

Source:https://data.worldbank.org/indicator/ST.INT.XPND.MP.ZS?view=chart

Table 3. The estimated model (8): Lower middle-income countries, 1995-2018.

Lower middle-income countries	R = 0.831127 Variance explained: 69.101%		
		π	γ
	Estimate	1.217294	0.234344
	Std.Err.	0.127489	0.145221
	t(21)	9.548255	1.613702
	p-level	0.00000	0.121519

The expenditures moved from 3.8661 to 5.4868 (unit of measure: % of total imports) in the period 1995-2018 in lower-middle-income countries. According to the logistic equation, for $1 < \pi < 2$ the expenditures moved to $z = (\pi - 1) / \pi$, or $(1.217294 - 1) / 1.217294 = 0.1785$. According to (7.), the equilibrium value of the expenditures was 0.1785 / (0.234344 / 1.217294) or 0.1785 / 0.1925 or 0.9273 or 5.0879 (unit of measure: % of total imports) in lower-middle-income countries in the observed period.
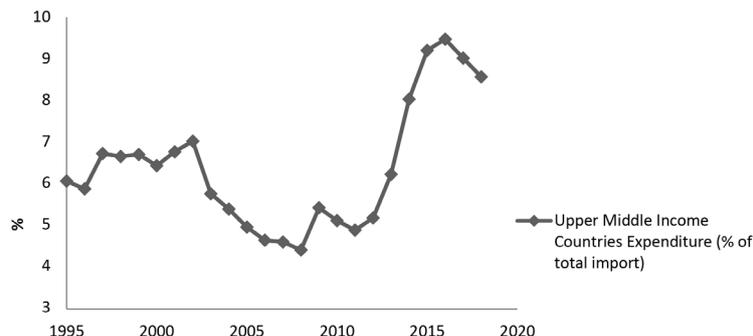

Figure 4. Middle-income countries, Expenditures (% of total imports).
Source: https://data.worldbank.org/indicator/ST.INT.XPND.MP.ZS?view=chart

Table 4. The estimated model (8): Middle-income countries, 1995-2018.

Middle-income countries	R = 0.89829 Variance explained: 80.623%		
		π	γ
	Estimate	1.07946	0.087817
	Std.Err.	0.09972	0.129405
	t(21)	10.82471	0.678620
	p-level	0.00000	0.504789

The expenditures moved from 5.5851 to 7.8151 (unit of measure: % of total imports) in the period 1995-2018 in middle-income countries. According to the logistic equation, for $1 < \pi < 2$ the expenditures moved to $z = (\pi - 1) / \pi$, or $(1.07946 - 1) / 1.07946 = 0.0736$. According to (7.), the equilibrium value of the expenditures was 0.0736 / (0.087817 / 1.07946) or 0.0736 / 0.0814 or 0.9042 or 7.6189 (unit of measure: % of total imports) in middle-income countries in the observed period.

Figure 5. Upper middle-income countries, Expenditures (% of total imports).
Source:https://data.worldbank.org/indicator/ST.INT.XPND.MP.ZS?view=chart

Table 5. The estimated model (8): Upper middle-income countries, 1995-2018.

	R = 0.46421 Variance explained: 21.549%		
		π	γ
	Estimate	1.144775	0.218628
	Std.Err.	0.281749	0.367056
	t(21)	4.063107	0.595627
	p-level	0.000559	0.557790

(Left vertical label: Upper middle-income countries)

The expenditures moved from 6.0463 to 8.5599 (unit of measure: % of total imports) in the period 1995-2018 in upper-middle-income countries. According to the logistic equation, for $1 < \pi < 2$ the expenditures moved to z = (π – 1) / π , or (1.144775 – 1) / 1.144775 = 0.1265. According to (7.), the equilibrium value of the expenditures was 0.1265 / (0.218628 / 1.144775) or 0.1265 / 0.191 or 0.6623 or 6.2701 (unit of measure: % of total imports) in upper-middle-income countries in the observed period.

4. THE CHAOTIC INTERNATIONAL TOURISM RECEIPTS GROWTH MODEL

International tourism receipts (R) are expenditures by international inbound visitors, including payments to national carriers for international transport. They include any other prepayment made for goods or services received in the destination country. They also may include receipts from same-day visitors, except when these are important enough to justify separate classification. Their share in exports (r) is calculated as a ratio to exports of goods and services (E), which comprise all transactions between residents of a country and the rest of the world involving a change of ownership from residents to nonresidents of general merchandise, goods sent for processing and repairs, nonmonetary gold, and services (www.worldbank.org).

$$R = R / E \tag{9}$$

We index r by t, i.e., write r_t to refer to the size at time steps t = 0,1,2,3,...Now the growth rate is measured by the quantity already given corresponding to the expression:

$$\frac{r_{t+1} - r_t}{r_t} \tag{10}$$

It is postulated that the growth rate of the ratio (r) between international tourism receipts and exports at time t should be proportional to $1 - r_t$, after introducing a suitable parameter δ, i.e.,

$$\frac{r_{t+1} - r_t}{r_t} = \delta (1 - r_t) \tag{11}$$

Solving the last equation yields the unemployment rate growth model, i.e.,

$$r_{t+1} = r_t + \delta r_t (1 - r_t) \tag{12}$$

or

$$r_{t+1} = (1 + \delta) r_t - \delta r_t^2 \tag{13}$$

This model given by equation (13) is called the logistic model. For most choices of δ, there is no explicit solution for (13). This difference equation (13) will possess a chaotic region.

Also, it is possible to show that iteration process for the logistic equation:

$$z_{t+1} = \pi \, z_t \, (1 - z_t), \; \pi \in [0, 4], \; z_t \in [0, 1] \tag{6}$$

is equivalent to the iteration of the growth model (13) when we use the identification

$$z_t = \frac{\delta}{1+\delta} \, r_t \; \text{ and } \; \pi = 1+\delta \tag{14}$$

Using (13) and (14) we obtain:

$$z_{t+1} = \frac{\delta}{1+\delta} \, r_{t+1} = \frac{\delta}{1+\delta} \, [(1 + \delta) \, r_t - \delta \, r_t^2] = \delta \, r_t - \frac{\delta^2}{1+\delta} \, r_t^2$$

Using (6) and (14) we obtain:

$$z_{t+1} = \pi \, z_t \, (1 - z_t) = (1 + \delta) \frac{\delta}{1+\delta} \, r_t \, (1 - \frac{\delta}{1+\delta} \, r_t) = \delta r_t - \frac{\delta^2}{1+\delta} \, r_t^2$$

Thus we have that iterating (13) is the same as iterating (6) using (14).

5. EMPIRICAL EVIDENCE

The main aim of this paper is to analyze the local growth stability of the ratio of international tourism receipts (R) to exports of goods and services (E) in the period 1995-2018. in: high-income countries, low & middle-income countries, lower-middle-income countries, middle-income countries, and upper-middle-income countries (see Figures 6-10). In this sense, it is important to use the logistic model (15):

$$r_{t+1} = \pi \, r_t - \delta \, e_t^2 \tag{15}$$

where:
   r - the international tourism receipts (as the ratio to exports of goods and services),
   π - $(1+\delta)$, δ - the constant of proportionality.

Now, the model (15) is estimated (see Tables 6-10.).
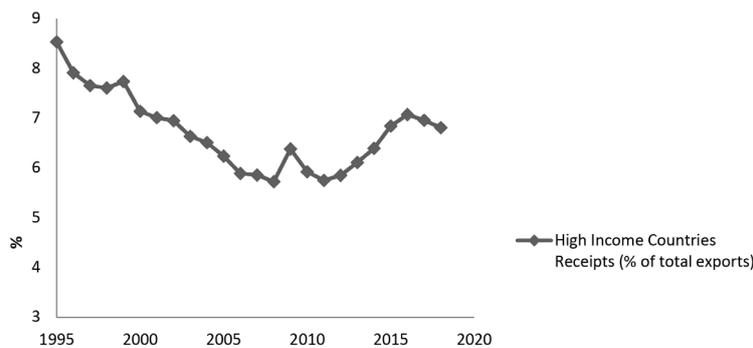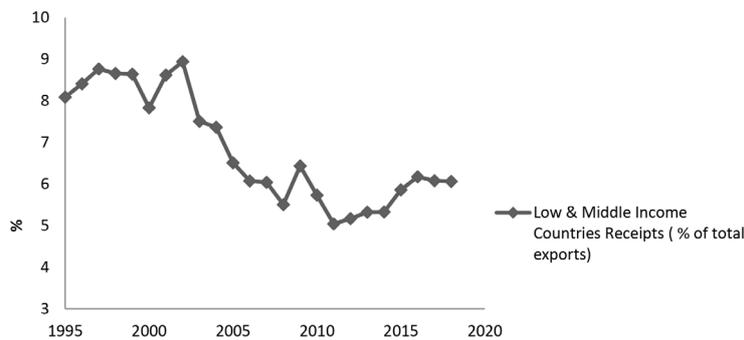

Figure 6. High-income countries, Receipts (% of total exports).
Source: https://data.worldbank.org/indicator/ST.INT.RCPT.XP.ZS?view=chart

Table 6. The estimated model (15): High-income countries, 1995-2018.

		R = 0.91797 Variance explained: 84.268%	
High-income countries		π	γ
	Estimate	1.17528	0.233532
	Std.Err.	0.17395	0.090804
	t(21)	15.89264	2.571812
	p-level	0.00000	0.017778

The receipts moved between 6.0463 and 8.5266 (unit of measure: % of total exports) in the period 1995-2018 in high-income countries. According to the logistic equation, for $1 < \pi < 2$ the receipts moved to $z = (\pi - 1) / \pi$, or $(1.17528 - 1) / 1.17528 = 0.1491$. According to (14.), the equilibrium value of the expenditures was $0.1491 / (0.233532 / 1.17528)$ or $0.1491 / 0.1987$ or 0.7504 or 6.3984 (unit of measure: % of total exports) in high-income countries in the observed period.
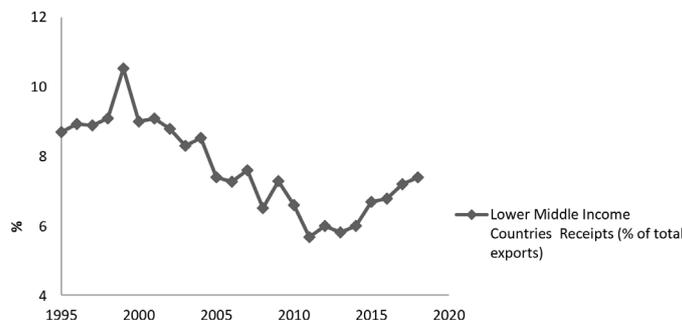

Figure 7. Low & middle-income countries, Receipts (% of total exports).
Source: https://data.worldbank.org/indicator/ST.INT.RCPT.XP.ZS?view=chart

Table 7. The estimated model (15): Low & middle-income countries, 1995-2018.

		R = 0.91255 Variance explained: 83.275%	
Low & middle-income countries		π	γ
	Estimate	1.06888	0.102672
	Std.Err.	0.09616	0.114700
	t(21)	11.11570	0.895135
	p-level	0.00000	0.380861

The receipts moved from 8.0903 to 6.0655 (unit of measure: % of total exports) in the period 1995-2018 in low & middle income countries. According to the logistic equation, for $1 < \pi < 2$ the receipts moved to $z = (\pi - 1) / \pi$, or $(1.06888 - 1) / 1.06888 = 0.0644$. According to (14.), the equilibrium value of the expenditures was $0.0644 / (0.102672 / 1.06888)$ or $0.0644 / 0.0961$ or 0.6701 or 5.9928 (unit of measure: % of total exports) in low & middle income countries in the observed period.
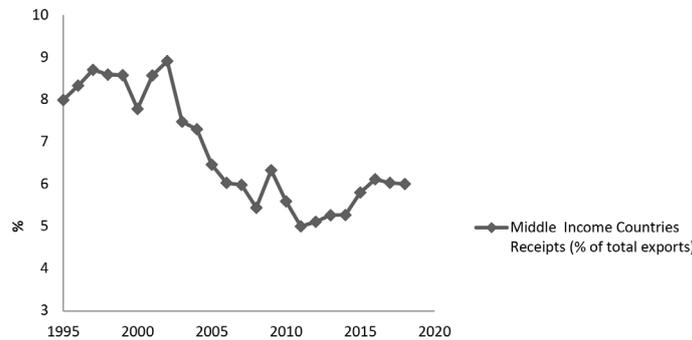

Figure 8. Lower middle-income countries, Receipts (% of total exports).
Source: https://data.worldbank.org/indicator/ST.INT.RCPT.XP.ZS?view=chart

Table 8. The estimated model (15): Lower middle-income countries, 1995-2018.

Lower middle-income countries	R = 0.86711 Variance explained: 75.188%		
		π	γ
	Estimate	1.14345	0.200432
	Std.Err.	0.11413	0.146230
	t(21)	10.01858	1.370659
	p-level	0.00000	0.184955

The receipts moved from 8.7120 to 7.4149 (unit of measure: % of total exports) in the period 1995-2018 in lower-middle-income countries. According to the logistic equation, for $1 < \pi < 2$ the receipts moved to $z = (\pi - 1) / \pi$, or $(1.14345 - 1) / 1.14345 = 0.1255$. According to (14), the equilibrium value of the expenditures was 0.1255 / (0.200432/1.14345) or 0.1255 / 0.1753 or 0.7159 or 7.5422 (unit of measure: % of total exports) in lower middle-income countries in the observed period.
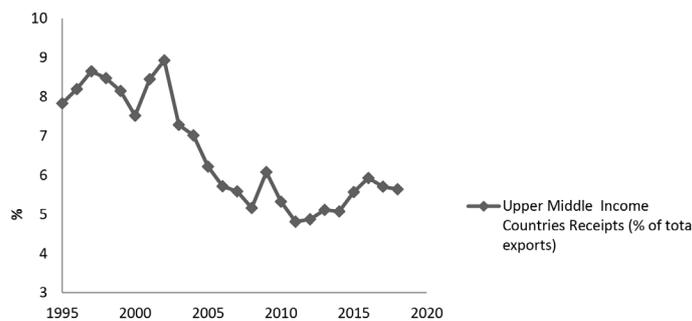

Figure 9. Middle-income countries, Receipts (% of total exports).
Source: https://data.worldbank.org/indicator/ST.INT.RCPT.XP.ZS?view=chart

Table 9. The estimated model (15): Middle-income countries, 1995-2018.

Middle-income countries	R = 0.47489 Variance explained: 22.552%		
		π	γ
	Estimate	1.06727	0.101047
	Std.Err.	0.09536	0.114224
	t(21)	11.19247	0.884637
	p-level	0.00000	0.386370

The receipts moved from 7.9876 and 5.9977 (unit of measure: % of total exports) in the period 1995-2018 in middle income countries. According to the logistic equation, for $1 < \pi < 2$ the receipts moved to $z = (\pi - 1) / \pi$, or $(1.06727 - 1)/ 1.06727 = 0.063$. According to (14), the equilibrium value of the expenditures was 0.063 / (0.101047 / 1.06727) or 0.063 / 0.0947 or 0.6653 or 5.927 (unit of measure: % of total exports) in middle-income countries in the observed period.

Figure 10. Upper middle-income countries, Receipts (% of total exports).
Source: https://data.worldbank.org/indicator/ST.INT.RCPT.XP.ZS?view=chart

Table 10. The estimated model (15): Upper-middle-income countries, 1995-2018.

Upper-middle-income countries	R = 0.90943 Variance explained: 82.707%		
		π	γ
	Estimate	1.07053	0.110353
	Std.Err.	0.09825	0.120360
	t(21)	10.89623	0.916863
	p-level	0.00000	0.369624

The receipts moved from 7.8344 to 5.6439 (unit of measure: % of total exports) in the period 1995-2018 in upper middle-income countries. According to the logistic equation, for $1 < \pi < 2$ the receipts moved to $z = (\pi - 1) / \pi$, or $(1.07053 - 1) / 1.07053 = 0.0659$. According to (14), the equilibrium value of the expenditures was $0.0659 / (0.110353 / 1.07053)$ or $0.0659 / 0.1031$ or 0.6392 or 5.7091 (unit of measure: % of total exports) in upper middle-income countries.

CONCLUSION

This paper confirms the stable movement of the international tourism expenditure (as the ratio to imports of goods and services), in high-income countries, low & middle-income countries, lower-middle-income countries, middle-income countries, and upper-middle-income countries in the period 1995-2018.

Also, this paper confirms the stable movement of the international tourism receipts (as the ratio to exports of goods and services) in high-income countries, low & middle-income countries, lower-middle-income countries, middle-income countries, and upper-middle-income countries in the period 1995-2018.

REFERENCES

Benhabib, J., Day, R.H. (1981) Rational Choice and Erratic Behaviour, Review of Economic Studies 48: 459-471.

Benhabib, J., Day, R.H. (1982) Characterization of Erratic Dynamics in the Overlapping Generation Model. Journal of Economic Dynamics and Control 4: 37-55.

Day, R.H. (1982) Irregular Growth Cycles. American Economic Review 72: 406-414.

Day, R.H. (1983) The Emergence of Chaos from Classica Economic Growth. Quarterly Journal of Economics 98: 200-213.

Day, R.H. (1997) Complex Economic Dynamics. Volume I: "An introduction to dynamical systems and market mechanism", MIT Press, In: Discrete Dynamics in Nature and Society, Vol. 1. 177-178.

Goodwin, R.M. (1990) Chaotic Economic Dynamics. Clarendon Press, Oxford.

Grandmont, J.M. (1985) On Endogenous Competitive Business Cycles. Econometrica 53: 994-1045.

Jablanovic, V. (2019) The Chaotic Growth Model of the Total Consumption Visitor Expenditure: Euro Area. 4th International Thematic Monograph Modern management tools and of economy of tourism sector in present era. pp. 17-31, Association of Economists and Managers of the Balkans in cooperation with the Faculty of Tourism and Hospitality, Ohrid, North Macedonia, ISBN 978-86-80194-29-5, DOI https://doi.org/10.31410/tmt.2019.17

Medio, A. (1996) Chaotic dynamics. Theory and applications to economics. Cambridge University Press, In: De Economist 144 (4), 695-698. Covid-19 and Tourism: Assessing the Economic Consequences (2020) United Nations Conference on Trade and Development. https://unctad.org/system/files/official-document/ditcinf2020d3_en.pdfhttps://data.worldbank.org/indicator/ST.INT.RCPT.XP.ZS?view=chart